Antenna Subset Selection with Bit Allocation for Multicarrier Spatial Diversity Transceivers

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Abstract – In this paper, two novel algorithms are presented that simultaneously perform bit allocation and antenna subset selection for multicarrier spatial diversity transceivers. The goal of the bit allocation is to increase overall system throughput while ensuring that the system remains below a specified error rate. One of the proposed algorithms uses the same signal constellation across all subcarriers, while the other varies the signal constellation. To reduce hardware costs, power consumption, and complexity, the proposed algorithms also employ antenna subset selection and a reduced set of radio frequency (RF) chains. However, unlike previously published algorithms, the proposed algorithms choose array configurations that vary across the subcarriers, yielding even further increases in the overall throughput. The results show that the proposed algorithms employing both multiple antennas and bit allocation will increase the overall throughput of the system. Furthermore, the reduced hardware costs due to a smaller number of RF chains is achieved with a negligible throughput penalty.

Keywords: Multicarrier Modulation, Bit Allocation, Spatial Diversity, Antenna Selection

1 Introduction

To enable high data rate wireless access, wireless transceivers are employing multiple transmit/receive antennas and multicarrier modulation [1]. Multiple antennas configured for spatial diversity can enhance the error robustness of the system, while multicarrier modulation can facilitate high-speed wireless data transmission by transforming a frequency-selective fading channel into a collection of approximately flat subchannels. The combination of multicarrier modulation and multiple antennas, configured for spatial diversity, has been shown to yield performance gains greater than the gains offered by each individual technology [2–4]. However, there exists the potential for further enhancements in performance and implementation.

Multicarrier systems can substantially increase throughput by tailoring the subcarrier signal constellation sizes according to the channel environment via a technique called *bit allocation*, which is applicable to either single antenna [5] or multiple antenna systems [6, 7]. Furthermore, to reduce hardware costs, power consumption, and complexity, a transceiver can be implemented with fewer RF chains than antennas. This results in the system activating only a subset of antennas, choosing the ones that can yield the best possible system performance. This technique is known as *antenna subset selection* [4, 8].

The combination of bit allocation and antenna subset selection would be advantageous to a multicarrier system employing multiple antennas, since it would incorporate the benefits

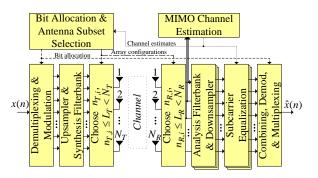


Fig. 1 Schematic of a multicarrier spatial diversity system employing bit loading and antenna subset selection.

offered by both technologies. For instance, it was shown that a transceiver employing algorithms performing both bit allocation and antenna subset selection achieved substantial throughput gains [9], even when using a reduced number of transmit and receive radio frequency (RF) chains [10]. Nevertheless, it is still possible to achieve a further increase in the overall throughput with a reduced set of RF chains by performing antenna subset selection on a per-subcarrier basis¹, while suffering a penalty in algorithm computational complexity.

In this paper, we present two novel algorithms that simultaneously perform bit allocation and per-subcarrier antenna subset selection for multicarrier spatial diversity transceiver, given a reduced number of RF chains. The bit allocation process is based on an efficient algorithm that quickly provides solutions close to the optimal allocation [5]. The different between the two proposed algorithms is that one employs uniform² bit allocation while the other performs non-uniform³ bit allocation. After a brief description of the system implementation in Section 2, the proposed algorithms are presented in Section 3. Simulation results are shown in Section 4 while several concluding remarks are made in Section 5.

2 System Implementation

A schematic of the multicarrier transceiver employing multiple antennas is shown in Fig. 1. The high-speed data stream x(n) is demultiplexed into N parallel data streams of different rates, and each stream is modulated using one of M supported

¹Although several algorithms performing bit allocation and subcarrierlevel antenna subset selection have been proposed [9], they only considered systems with a full set of RF chains available. As for algorithms applied to transceivers with a reduced set of RF chains [10], the antenna subset selection was only performed on the entire multicarrier signal rather than per-subcarrier.

²The same signal constellation is employed across all subcarriers.

³The subcarrier signal constellations vary across all subcarriers.

signal constellations. The choice of subcarrier signal constellation and data rate is determined by the proposed algorithms employed by the system (refer to Section 3 for details). These streams are then upsampled by a sampling factor N and filtered by one of N bandpass filters constituting the synthesis filterbank.

The proposed algorithms then decide which subcarrier outputs of the synthesis filterbank to summed together for each transmit antenna, given the multiple-input multiple-output (MIMO) channel conditions. The summed outputs are then sent to the corresponding transmit antennas, where simple transmit diversity techniques are employed across a subset of transmit antennas per subcarrier⁴. Note that each sum of outputs per transmit antenna may contain a different group of subcarriers. In other words, the synthesis filterbank output of subcarrier *i* is fed to $n_{T,i}$ transmit antennas, where $n_{T,i} \leq L_T < N_T$, while N_T and L_T are the number of available transmit antennas and RF chains⁵. The choice of the $n_{T,i}$ transmit antennas is based on whether the antenna configuration yields the largest overall throughput. Similarly, the proposed algorithms perform receive antenna subset selection for subcarrier i by choosing $n_{R,i}$ receive antennas yielding the largest overall throughput, where $n_{R,i}$ is constrained to be $n_R \leq L_R < N_R$, given that N_R and L_R are the number of available receive antennas and RF chains.

Each analysis filterbank, corresponding to one of the $n_{R,i}$ antennas, separates its intercepted signal into N subcarriers and downsamples it by a sampling factor N. Equalization is then performed per subcarrier to remove the distortion introduced to the transmitted signals by the MIMO channel. The equalized subcarrier signals are then combined, e.g., maximum ratio combining (MRC), followed by demodulation, and multiplexing to form the reconstructed high-speed data stream $\hat{x}(n)$. In the next section, the proposed algorithms used to determine the antenna subsets and the bit allocations will be presented.

3 Algorithm Descriptions

The primary objective of the proposed algorithms is to increase the overall throughput of the system while ensuring the mean BER, \bar{P} , is below a specified mean BER limit, P_T . This can be expressed mathematically as the following optimization problem:

$$\max_{s_i, b_i} \sum_{i=1}^N b_i \tag{1}$$

subject to :

$$\bar{P} = \left(\sum_{i=1}^{N} b_i P_i\right) \middle/ \left(\sum_{i=1}^{N} b_i\right) \le P_T, \quad (2)$$

where b_i is the number of bits per symbol for subcarrier *i*, P_i is the BER for subcarrier *i*, which is computed from the subcarrier signal-to-noise ratio (SNR), γ_i , via closed form ex-

- 1. Initialization: Compute P_i , $i = 1, \ldots, N$, for all M modulation schemes and all possible antenna configurations. Choose initial values of \hat{P} and δ for the iterative algorithm, where \hat{P} is the peak BER limit per subcarrier, and δ is the stepsize.
- 2. If the largest P_i for the largest available signal constellation is less than P_T , set all subcarriers to that constellation, employ 1 transmit antenna, employ 1 receive antenna, and exit algorithm, else go to Step 3.
- 3. If smallest P_i for the smallest (non-zero) signal constellation is greater than P_T , turn off all subcarriers and exit algorithm, else proceed to Step 4.
- 4. Select the subcarrier antenna configurations and signal constellations that yield the largest overall throughput while satisfying $P_i < \hat{P}$.
- 5. Compute \bar{P} using Eq. (2).
- 6. If $\bar{P} < P_T$, let $\hat{P} = \hat{P} + \delta$, else $\hat{P} = \hat{P} \delta$.
- 7. Select the subcarrier antenna configurations and signal constellations that yield the largest overall throughput while satisfying $P_i < \hat{P}$.
- 8. Compute $ar{P}'$, the new value of the mean BER.
- 9. If both $\bar{P} > P_T$ and $\bar{P}' > P_T$ (resp. $\bar{P} \leq P_T$ and $\bar{P}' \leq P_T$), and no previous straddling of P_T , let $\bar{P} = \bar{P}'$, $\hat{P} = \hat{P} \delta$ (resp. $\hat{P} = \hat{P} + \delta$), and go to Step 7, else go to Step 10.
- 10. If both $\bar{P} \leq P_T$ and $\bar{P}' \leq P_T$, and P_T was straddled before, let $\bar{P} = \bar{P}'$, $\hat{P} = \hat{P} + \delta$, and go to Step 7, else go to Step 11.
- 11. If both \bar{P} and \bar{P}' are straddling P_T and the number of times this occurred is less than a specified amount β , reduce δ , let $\bar{P} = \min(\bar{P}, \bar{P}')$, set $\hat{P} = \hat{P} \pm \delta$ (the future \bar{P}' should be on the same side of P_T as \bar{P}), and go to Step 7. Otherwise, finalize the allocation and end the algorithm.

Fig. 2 Proposed algorithm framework employing both bit allocation and subcarrier-level antenna subset selection.

pressions [11]⁶. The value of γ_i is equal to the composite SNR value due to the recombining of the different signal paths from the transmitter to the receiver. Thus, γ_i is also a function of the subcarrier antenna configuration $s_i, s_i \in S_{\text{config}}$, where the set S_{config} contains all possible transmit/receive antenna configurations⁷. Note that the constraint for the subcarrier antenna configurations being equal, i.e.:

$$s_1 = s_2 = \dots = s_N,\tag{3}$$

is not imposed in this work.

The secondary objective of the proposed algorithms is to activate as few antennas as possible. Suppose S_{\max} denotes the set of subcarrier transmit/receive antenna configurations that yield the largest throughput in Eq. (1), where $S_{\max} \subseteq S_{\text{config.}}$. Thus, the process of finding subcarrier configurations employing the fewest antennas can also be formulated as the optimization problem:

$$\min_{s_i \in \mathcal{S}_{\max}} (\mu_T \cdot n_{T,i}(s_i) + \mu_R \cdot n_{R,i}(s_i)) \tag{4}$$

where $0 < n_{T,i}(s) \le L_T$ and $0 < n_{R,i}(s) \le L_R$ are the number of active transmit and receive antennas for subcarrier

⁴The total transmit power level across all the transmit antennas per subcarrier is equivalent for all subcarrier transmit antenna array configurations. If there is more than one active transmit antenna in a given subcarrier configuration, the total power per subcarrier is divided evenly between the active antennas.

⁵In this work, we only consider the case when there are using fewer RF chains than available antennas.

⁶In a practical implementation, the BER values would be stored in a lookup table.

⁷The largest transmit (receive) antenna array size in $\mathcal{S}_{\text{config}}$ is constrained by the number of available transmit (receive) RF chains, L_T (L_R), which may be fewer than the number of available transmit (receive) antennas.

- 1. Commence search with the signal constellation employing the largest number of constellation points.
- 2. For subcarrier i, find all subcarrier antenna configurations that satisfy the condition $P_i < \hat{P}$, for $i=1,\ldots,N$.
- 3. Given these subcarrier antenna configurations, find those configurations that at least exclude the same $(N_T L_T)$ transmit and $(N_R L_R)$ receive antennas across all subcarriers.
- 4. If there does not exist a common excluded set antennas across all subcarriers, redo search with the signal constellation employing the next largest number of constellation points and proceed to Step 2. Otherwise, finalize subcarrier antenna configurations and exit the search.

Fig. 3 Proposed search routine for obtaining the subcarrier antenna configurations and uniform bit allocation when $L_T < N_T$ and/or $L_R < N_R$. This routine is employed in Steps 4 and 7 of Fig. 2.

antenna configuration s_i , and both μ_T and μ_R are weights⁸ satisfying the constraint $\mu_T + \mu_R = 1$.

Given these objectives, the proposed algorithms are designed to iteratively search for the bit allocation and subcarrier antenna configurations that yield the largest possible throughput while activating as few antennas as needed. The algorithm framework is shown in Fig. 2. Although this framework is similar to several previous algorithms [9, 10], there is a substantial difference. This difference is with respect to defining different subcarrier antenna configurations, i.e., Eq. (3), while operating with a reduced set of RF chains, i.e., $L_T < N_T$ and $L_R < N_R$. While the previous algorithms have done either one, they have never attempted to do both simultaneously. Therefore, the proposed algorithms impose the additional constraint that all chosen subcarrier antenna configurations only have access to the same L_T transmit and L_R receive antennas. The choice of these antennas, which are decided along with the bit allocation and subcarrier antenna configurations in Steps 4 and 7 of Fig. 2, are discussed in Sections 3.1 and 3.2, for the cases when uniform and non-uniform bit allocation are performed⁹.

3.1 Uniform Bit Allocation with Subcarrier-level Antenna Subset Selection

The proposed algorithm performing uniform bit allocation imposes the following constraint on the bit allocation:

$$b_1 = b_2 = \dots = b_N,\tag{5}$$

such that the signal constellations across all subcarriers are identical¹⁰. Thus, the search for both a bit allocation and a set of subcarrier antenna configurations is greatly simplified. The search routine for the proposed algorithm employing uniform bit allocation is shown in Fig. 3, which is employed at Steps 4 and 7 of Fig. 2.

The search routine operates by starting with the largest supported signal constellation and determining which subcarrier antenna configurations satisfy the condition $P_i < \hat{P}$, for i = 1, ..., N. Given these configurations, the routine then

1.	For subcarrier i , find the subcarrier antenna configuration(s) yielding the largest signal constellation that satisfies $P_i < \hat{P}$, for $i=1,\ldots,N$.
2.	Given these subcarrier antenna configurations, choose the L_T transmit and L_R receive antennas that occur most frequently in this set of configurations.
3.	Restricting the subcarrier antenna configurations to those that only use the subsets of transmit and receive antennas in Step 2, find configurations supporting the largest subcarrier signal constellation satisfying $P_i < \hat{P}$, for $i = 1, \ldots, N$.
4.	Upon completion of the search, finalize the subcarrier antenna configurations and signal constellations, and then exit the search.

Fig. 4 Proposed search routine for obtaining the subcarrier antenna configurations and non-uniform bit allocation when $L_T < N_T$ and/or $L_R < N_R$. This routine is employed in Steps 4 and 7 of Fig. 2.

determines if there consists a set of $(N_T - L_T)$ transmit and $(N_R - L_R)$ receive antennas that could deactivated for all subcarriers, and still yield an antenna configuration per subcarrier capable of supporting the signal constellation. If so, those antennas are deactivated and the remaining antennas are allocated the L_T transmit and L_R receive RF chains. If not, the process is repeated for the next largest signal constellation until either a solution is found or all the subcarriers are turned off, i.e., *nulled*.

Although less computationally complex, the proposed algorithm sacrifices some of the flexibility and throughput gains associated with multicarrier modulation. In the following subsection, the other proposed algorithm is be presented, which does exploit this flexibility.

3.2 Non-uniform Bit Allocation with Subcarrier-level Antenna Subset Selection

When the proposed algorithm performs non-uniform bit allocation, it does not impose the constraint of Eq. (5), resulting in additional flexibility in the bit allocation process. However, the search for both a bit allocation and a set of subcarrier antenna configurations becomes substantially more computationally intensive. The search routine for the proposed algorithm employing non-uniform bit allocation is shown in Fig. 4, and is employed at Steps 4 and 7 of Fig. 2

This search routine starts by searching for the subcarrier antenna configurations that yield the largest signal constellation per subcarrier that satisfies $P_i < \hat{P}$, for i = 1, ..., N. Once the antenna configurations have been obtained, the routine assesses how often a specific antenna is activated, across all subcarriers and for each transmit and receive antenna. The algorithm then chooses the L_T transmit and L_R receive antennas that have the highest frequency of occurrence. Once the selected antennas have been chosen for activation, the routine searches for subcarrier antenna configurations that support the largest subcarrier signal constellation, which satisfies $P_i < \hat{P}$, for i = 1, ..., N. Upon completion of the search, the routine finalizes the bit allocation and the configurations before exiting.

Although more flexible than the previous algorithm, this proposed algorithm has a substantially higher computational complexity. Thus, the two proposed algorithms highlight the trade-offs between algorithm computational complexity and

⁸Minimizing the number of transmit and receive antennas is assumed equally important in this work, hence these weights are set to $\mu_T = \mu_R = 0.5$.

⁹The difference between the two proposed algorithms is with respect to how they perform the bit allocation.

¹⁰This form of bit allocation is available in current standards [12].

throughput, as will be illustrated in the next section.

4 Simulation Results

To evaluate the performance of the proposed algorithms, a multicarrier system based on some of the operating parameters used in the IEEE 802.11a standard are employed [12]¹¹. The option to null subcarriers also exists in circumstances where the prevailing channel conditions are too poor. Results from all the proposed algorithms were obtained for a target mean BER value of $P_T = 10^{-5}$. Moreover, the limit $\beta = 10$ is used in Step 11 of Fig. 2 since this gives the algorithm enough flexibility to zoom in to the final configuration.

The antenna elements employed by the arrays are $\lambda/2$ omnidirectional dipole antennas¹² placed in a uniformly-spaced linear array with adjacent antenna separation of d and oriented such that they are all perpendicular to the xy-plane, i.e., vertically polarized. The largest array configuration studied in this work possessed 3 antennas, although the proposed algorithms could be employed in systems with larger array sizes. Simple transmit diversity is employed at the transmitter while maximal ratio combining (MRC) is performed at the receiver to recombine the received signals. The physical separation between the transmitter and receiver was varied between 1 m and 60 m¹³. The MIMO channel consists of a collection of frequency-selective fading SISO channel responses generated using the method proposed by Saleh and Valenzuela [13]¹⁴. The SISO components of the MIMO channel were assumed to be uncorrelated in most of this work since the adjacent antenna separation was set to $d = 5\lambda$. However, the effect of correlation due to antenna separation was also examined for $d = 0.25\lambda$ using the model proposed in [14]. Finally, for each MIMO channel realization, the algorithms were operating at 70 different averaged SNR values equally spaced in the logarithmic domain. The trials were repeated for 10000 different MIMO channel realizations and the results averaged.

Figs. 5 and 6 show the throughput results of a multicarrier diversity system employing the two proposed subcarrier-level (SC) algorithms. For comparison purposes, the signal-level (Sig) algorithms performing bit allocation and antenna subset selection have also been included [10]. Furthermore, one set of results was obtained when there was a full set of RF chains, i.e., $(L_T = N_T, L_R = N_R)$, while another set of results is for the case when the system employs a reduced set of RF chains, i.e., $(L_T < N_T, L_R < N_R)$. Several observations can be made regarding the relative throughput values of these results. First, the throughput difference between a system employing a full or reduced set of RF chains and using the proposed (SC) algorithm is on the order of 10 to 15 bits/symbol. For instance, at an SNR of 10 dB, the throughput difference between systems employing the proposed (SC) algorithm is 14 bits/symbol in Fig. 5, while in Fig. 6 the difference is 15 bits/symbol. These throughput differences are much larger relative to a system employing the (Sig) algorithms, which have throughput differences of 5 bits/symbol in

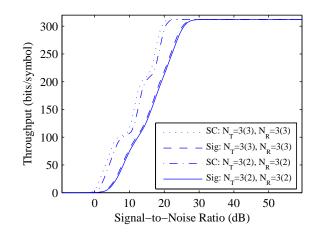


Fig. 5 Throughput results for multicarrier transceivers performing uniform bit allocation and employing either signal-level (Sig) or the proposed subcarrier-level (SC) antenna subset selection (number of available RF chains in brackets next to number of antennas).

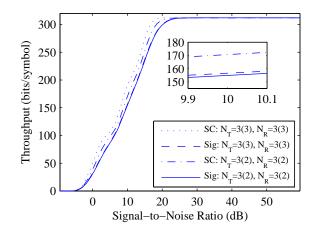


Fig. 6 Throughput results for multicarrier transceivers performing non-uniform bit allocation and employing either signal-level (Sig) or the proposed subcarrier-level (SC) antenna subset selection (number of available RF chains in brackets next to number of antennas).

Fig. 5 and 2 bits/symbol in Fig. 6. This is due to the additional flexibility afforded to the proposed (SC) algorithms by performing subcarrier-level antenna subset selection. However, considering the 33% savings in RF chains, a slightly larger throughput difference is justifiable. Second, employing the proposed (SC) algorithms achieves a throughput increase of approximately 22 bits/symbol relative to a system employing the (Sig) algorithms. Third, the throughput increase of a system employing the proposed (SC) algorithm with non-uniform bit allocation is substantial relative to a system employing the proposed (SC) algorithm with uniform bit allocation. For example, at 10 dB, the difference between systems with the same array configuration is about 70 bits/symbol.

The loss of throughput due to closely-spaced antennas is shown in Fig. 7. Most of the loss occurs around medium-range SNR values, where the proposed algorithms are trying to find the array configuration and bit allocation that yield the largest throughput. When the adjacent antenna spacing is 5λ , the loss in throughput is approximately 2 to 3 bits/symbol. However, if the adjacent antenna spacing is reduced to λ , the throughput substantially increases to a maximum of 15 bits/symbol.

¹¹The system possesses N = 64 subcarriers (6 "guard subcarriers" at each end of the 16.6 MHz bandwidth), uses BPSK, QPSK, square 16-QAM, and square 64-QAM modulation, and operates at a frequency of 5 GHz.

¹²The wavelength is equal to $\lambda = 1/(5 \text{ GHz}) = 0.06 \text{ m}.$

¹³The change in transmitter/receiver separation distance corresponds to an SNR change ranging from 59 dB to -11 dB.

¹⁴The SISO components were assumed to be time-invariant, non-line-ofsight, and uncorrelated.

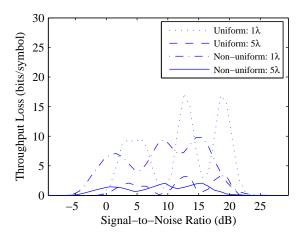


Fig. 7 Throughput loss due to correlation for multicarrier transceivers employing the proposed algorithms. Adjacent antenna spacings of λ and 5λ are employed for an antenna array configuration of $N_T = 3(L_T = 2)$, $N_R = 3(L_R = 2)$.

This is due to the correlation between them increases, which reduces the effectiveness of the spatial diversity. As a result, the flexibility of the algorithms to find an appropriate solution also decreases with the adjacent antenna spacing.

5 Conclusion

Two novel algorithms for multicarrier systems, employing multiple antennas in a spatial diversity configuration, have been presented. The algorithms perform bit allocation (either uniform or non-uniform) and antenna subset selection. Unlike previously published algorithms, the proposed algorithms perform the antenna subset selection per subcarrier, with each subcarrier possibly having a different active antenna configuration. The results exhibit a throughput increase when the proposed algorithms are employed by the system relative to the previously published algorithms. Moreover, with subcarrierlevel antenna subset selection, the throughput performance remains almost the same when two thirds of the RF chains are available, which translates into a hardware savings of 33%. Finally, non-uniform bit allocation yields a larger throughput increase at the cost of an increase in computational complexity.

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